Can you see it?

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1 Legendre Polynomials

The following definition of Legendre polynomials is taken from mathworld[1].

The Legendre polynomials $P_n(x)$ can be defined by the contour integral

$$P_n(z) = \frac{1}{2\pi\ i} \oint (1 - 2tz + t^2)^{-1/2}$$

where the contour encloses the origin and is traversed in a counterclockwise direction.

The first few Legendre polynomials are

$$P_0(x) = 1$$

$$P_1(x) = x$$

$$P_2(x) = \frac{1}{2}(3x^2 - 1)$$

$$P_3(x) = \frac{1}{2}(5x^3 - 3x)$$

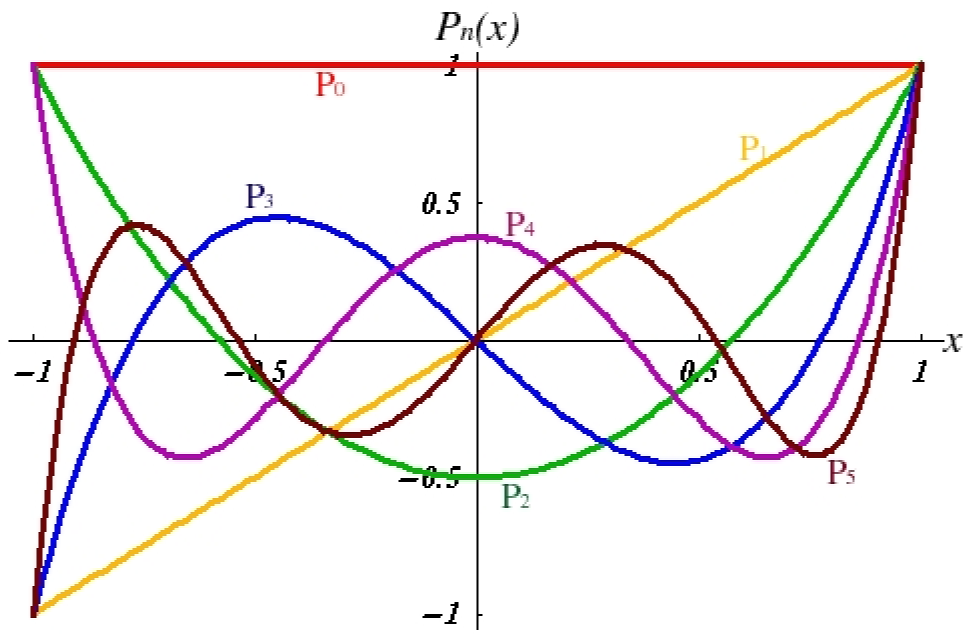
$$P_4(x) = \frac{1}{8}(35x^4 - 30x^2 + 3)$$

$$P_5(x) = \frac{1}{8}(63x^5 - 70x^3 + 15x)$$

$$P_6(x) = \frac{1}{16}(231x^6 - 315x^4 + 105x^2 - 5).$$

Below is a picture of the Legendre polynomials for $x \in [-1, 1]$ and $n = 0, 1, 2, 3, 4, 5$.

[1] http://mathworld.wolfram.com/LegendrePolynomial.html



One day while looking at a similar picture in a numerical methods book, I noticed that

$$\int_{-1}^{1} P_n(x)dx = 0$$

for $n \in Z$ where $n > 0$. Can you see how the area bounded by the x axis and each curve between -1 and 1 is equal?